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Conductive length in heat exchange problems of shells and truss constructions

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Abstract—Analysis of the conductance influence on the temperature regimes of thermally thin shells and truss constructions is carried out. For the spacial limit of the conductance influence the conductive length is introduced. For the evaluation of the conductive influence the criterion presented by the ratio of the characteristic dimension of the shell to the conductive length is proposed. The conductance can be neglected when this criterion is much larger than unity. When this relation is not fulfilled the conductance contribution is considerable. The evaluation of this criterion on the stage of the problem formulation allow determination of the need of the conductance in the heat transfer models. As an example of the application of the introduced criterion the development of the heat transfer computational models for airship shells, radio telescopes shells and truss constructions could be mentioned.

1. INTRODUCTION

DEVELOPMENT of many constructions needs the calculations of temperature fields in thermally thin shells and truss constructions under the radiative–convective heat transfer conditions [1–4]. As usual these problems belong to the area of complex heat transfer and in the stage of models formulations it is necessary to take into account all the three modes of the heat transfer: radiation; convection; and conductance. In the general case the temperature fields of the thermally thin shells and truss constructions can be described by the following differential equation which includes the surface heat exchange conditions in the form of equivalent energy sources distributions:

$$\delta\rho c\frac{\partial T}{\partial t} = \delta k\nabla^2 T - h_1(T-T_{\alpha1}) - h_2(T-T_{\alpha2}) - (\varepsilon_1+\varepsilon_2)\sigma T^4 + \Sigma As_i I_i \quad (1)$$

where h_1, h_2 are the convective heat transfer coefficients on both surfaces of the shell, $T_{\alpha1}$, $T_{\alpha2}$ are the temperature of the convective flows near both surfaces of the shell, ε_1, ε_2 are the emissivities of the coatings for both surfaces of the shell, I_i are the different radiation flux densities and As_i are the corresponding absorptivities of the coatings.

Equation (1) describes the temperature fields in the shells when the corresponding Biot numbers are small:

$$Bi = \frac{(h_1+h_2)\delta}{k} \ll 1 \quad (2)$$

and the cross-sectional temperature difference can be neglected.

In the case when the thermal time constant of the shell:

$$t_{sh} = \delta\rho c/(h_1+h_2) \quad (3)$$

is much smaller than the times of variations in heat exchange conditions the time derivative in equation (1) can be neglected. In this case the temperature field follows the time-dependent heat exchange conditions without noticeable delays and can be described by the stationary equation as:

$$\delta k\nabla^2 T = h_1(T-T_{\alpha1}) + h_2(T-T_{\alpha2}) + (\varepsilon_1+\varepsilon_2)\sigma T^4 - \Sigma As_i I_i. \quad (4)$$

It is evident that under some operating conditions the conductive heat transfer can be neglected and equation (4) can be further reduced to the algebraic or integral one depending on the contribution of the radiative heat transfer components. The integral radiative component arises in equations (1), (4) when the radiative heat transfer between the elements of the shell is considerable [5].

This paper is devoted to the analysis of the conductance influence on the temperature fields in the shells and truss constructions and to the formulation of the criterion which could allow one to evaluate the contribution of conductive heat transfer at the stage of the formulation of heat exchange models.

2. MODEL STUDY

Let us consider the stationary one-dimensional temperature field for the case of stepwise radiation flux density on the surface of the sheet (Fig. 1(a)). The nonlinear radiative heat transfer component is supposed to be small in comparison with the convective heat transfer component. This model problem is described by the following differential equation:

NOMENCLATURE

As	absorptivity, dimensionless	t_{sh}	thermal time constant [s]
Bi	Biot number, dimensionless	T	temperature [K]
c	specific heat [J kg^{-1} K^{-1}]	T_α	temperature of the convective flow [K]
h	convective heat transfer coefficient [W m^{-2} K^{-1}]	T_0	initial temperature [K]
h_s	total heat transfer coefficient [W m^{-2} K^{-1}]	T_l	limit case temperature [K]
H	dimensionless complex	z	longitudinal independent variable [m].
I, I_0	radiation flux densities [W m^{-2}]		
k	conductance coefficient [W m^{-1} K^{-1}]		Greek symbols
L	characteristic dimension, length [m]	δ	thickness of the shell, characteristic cross-sectional dimension [m]
L_C	conductive length [m]	ε	emissivity, dimensionless
P	heat transfer perimeter of the element [m]	ζ	dimensionless longitudinal independent variable [m]
q_s	heat flux density on the surface [W m^{-2} K^{-1}]	θ	dimensionless temperature
q_s^*	dimensionless value of the heat flux density on the surface	θ_l	limit case dimensionless temperature
R	radius of the shell [m]	θ_0	initial dimensionless temperature
S	cross-sectional area of the element [m^2]	ρ	density [kg m^{-3}]
t	time variable [s]	σ	Stefan constant
		τ	dimensionless time variable
		φ	angle independent variable.

$$\delta k \frac{d^2 T}{dz^2} = h_s(T - T_\alpha^*) - AsI_0\chi(-z) \quad (5)$$

with the boundary conditions

$$z \to -\infty: \quad T = T_\alpha^* + AsI_0/h_s \quad (6)$$

$$z \to \infty: \quad T = T_\alpha^* \quad (7)$$

where

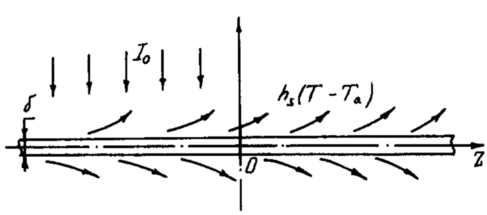

FIG. 1(a). Scheme for the model study.

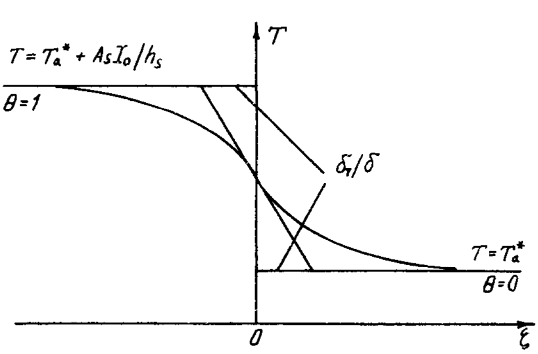

FIG. 1(b). Temperature distribution in the sheet and graphical interpretation of the conductive length.

$$\chi(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

is Heaviside's unit function modelling the stepwise change of the radiation flux density.

The solution of this model problem is given by

$$\theta(\zeta) = \begin{cases} 1 - 0.5\exp(\sqrt{Bi}\,\zeta) & \zeta < 0 \\ 0.5\exp(-\sqrt{Bi}\,\zeta) & \zeta \geq 0 \end{cases} \quad (8)$$

where $\theta = (T - T_\alpha^*)h_s/AsI_0$ is the dimensionless temperature, $\zeta = z/\delta$ is the dimensionless coordinate and $Bi = h_s\delta/k$ is the dimensionless Biot number.

This solution shows that the temperature distribution varies from 0 to 1 within the limited domain near the point of the stepwise increasing of the radiation flux density (Fig. 1(b)). For the dimensionless evaluation of the characteristic length of this domain the following approach can be used:

$$\frac{\delta_T}{\delta} = \frac{\theta|_0 - \theta|_l}{-\left.\dfrac{d\theta}{d\zeta}\right|_0}. \quad (9)$$

The geometrical interpretation of this expression is shown in Fig. 1(b). Substituting equation (8) in (9) one can show that the conductivity influence is extended from the zero point on the following length:

$$L_C = \left(\frac{k\delta}{h_s}\right)^{1/2} \quad (10)$$

which can be interpreted as the conductive length.

This length presents the spacial limit of the conductivity influence on the temperature fields in thermally thin shells. The heat transfer flux resulting from the temperature non-uniformity along the shell is dissipated by the heat transfer from the shell surfaces on the conductive length. In other words two arbitrary points of the shell possess a conductive interaction if the distance between them does not exceed the introduced conductive length. If the characteristic dimension of the shell is much larger than the conductive length, the conductivity influence cannot be taken into account and the differential component $k\nabla^2 T$ can be neglected in the heat transfer model. In this case the conductance affects the temperature field only along the lines of stepwise changes in heat exchange parameters on the shell surfaces. This influence has an effect only within the conductive length counted from the lines of stepwise changes of heat transfer parameters.

Thus, the need of the conductance mechanism in the heat transfer models of thermally thin shells can be considered on the basis of the following criterion, given by the ratio of the shell characteristic dimension to the introduced conductive length:

$$\frac{L}{L_C} = \frac{Lh_s^{1/2}}{(k\delta)^{1/2}}. \tag{11}$$

If the value of the criterion is much larger than unity $L/L_C \gg 1$ the conductivity contribution can be neglected. If this relation is not fulfilled the conductivity contribution should be taken into account. Note the case when this criterion is much smaller than the unity $L/L_C \ll 1$. In this case the conductivity contribution is so high that it would make uniform the temperature field of the shell held under non-uniform heat exchange conditions.

3. TEMPERATURE FIELDS OF SHELLS

3.1. *Formulation of the problem*

As an example of the conductance influence let us consider the temperature fields in the cylindrical and spherical shells. The contribution of the conductance in the temperature fields can be studied on the basis of the equation with the linear approximation of the heat transfer conditions:

$$\delta\rho c\frac{\partial T}{\partial t} = \delta k\nabla^2 T - h_s(T-T_\alpha^*) + q_s(\varphi) \tag{12}$$

where $q_s(\varphi)$ is the total surface heat flux distribution, $h_s = h_1+h_2$ is the total convective–radiative heat transfer coefficient of both the surfaces of the shell, $T_\alpha^* = (h_1T_{\alpha 1}+h_2T_{\alpha 2})/(h_1+h_2)$ is the reduced temperature of the convective flows near both the surfaces of the shell, ∇^2 is the Laplacian for the cylindrical and spherical shells, respectively, which is given by

$$\nabla^2 = \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}, \quad \nabla^2 = \frac{1}{R^2}\left(\frac{\partial^2}{\partial\varphi^2}+\text{ctg}\,\varphi\frac{\partial}{\partial\varphi}\right) \tag{13}$$

where R is the radius of the shells.

The boundary conditions of the problem are written as:

$$\left.\frac{\partial T}{\partial\varphi}\right|_{\varphi=0} = 0, \quad \left.\frac{\partial T}{\partial\varphi}\right|_{\varphi=\pi} = 0. \tag{14}$$

The initial state is given by:

$$T|_{t=0} = T_0. \tag{15}$$

Solution of equations (12)–(15) for both the shells can be obtained by means of eigenfunction expansion of the source function in equation (12) [6]. To present the solutions in compact form one can introduce the following dimensionless independent variables:

$$\varphi' = \varphi/\pi, \quad \tau = tk/\rho c(\pi R)^2 \tag{16}$$

and the dimensionless temperature:

$$\theta(\tau,\varphi') = \frac{(T-T_\alpha^*)h_s}{q_s(0)}. \tag{17}$$

3.2. *Cylindrical shell*

Using the technique of eigenfunction expansion of the source function in the conductivity equation one can obtain the following solution for the cylindrical shell:

$$\theta(\tau,\varphi') = \sum_{n=0}^{\infty}\left\{\frac{L^2/L_C^2}{\mu_n^2+L^2/L_C^2} + \left[\theta_{n0}-\frac{L^2/L_C^2}{\mu_n^2+L^2/L_C^2}Q_n\right]e^{-(\mu_n^2+L^2/L_C^2)\tau}\right\}\cos\mu_n\varphi' \tag{18}$$

where $\mu_n = 0,\ \pi(2n-1)/2\ (n = 1,\ldots,\infty)$ are the eigenvalues of the corresponding Sturm–Liouville problem

$$Q_n = \int_0^1 q_s^*(\varphi')\cos\mu_n\varphi'\,d\varphi' \Big/ \int_0^1 \cos^2\mu_n\varphi'\,d\varphi'$$

are the coefficients of the eigenfunction expansion of the dimensionless surface heat flux distribution $q_s^*(\varphi') = q_s(\varphi)/q_s(0)$

$$\theta_{n0} = \theta_0\int_0^1 \cos\mu_n\varphi'\,d\varphi' \Big/ \int_0^1 \cos^2\mu_n\varphi'\,d\varphi'$$

are the coefficients of the eigenfunction expansion of the dimensionless initial temperature $\theta_0 = (T_0-T_\alpha^*)h_s/q_s(0)$ and $L = \pi R$ is the characteristic dimension of the shell.

For the values $\tau \gg 1$ the expression (18) gives the stationary solution:

$$\theta(\varphi') = \int_0^1 q_s^*(\varphi')\,d\varphi' + \sum_{n=1}^{\infty} Q_n\frac{L^2/L_C^2}{\pi^2(2n-1)^2/4+L^2/L_C^2}\cos\left\{\frac{\pi}{2}(2n-1)\varphi'\right\} \tag{19}$$

where

$$Q_n = 2\int_0^1 q_s^*(\varphi')\cos\left\{\frac{\pi}{2}(2n-1)\varphi'\right\}\mathrm{d}\varphi'.$$

This solution has two components. The first one presents the mean value of the source function. In the limit $L/L_C \to 0$ the temperature field of the shell given by this term is uniform. The second term of the solution (19) is given by the infinite series dependent on the dimensionless criterion L/L_C and takes into account the contribution of the conductive heat transfer in the shell.

3.3. *Spherical shell*

Using the technique of the eigenfunction expansion of the source function in the conductivity equation one can obtain the following solution for the spherical shell:

$$\theta(\tau,\varphi') = \sum_{n=0}^{\infty}\left\{\frac{L^2/L_C^2}{\pi^2 n(n+1)+L^2/L_C^2}Q_n + \left[\theta_{n0} - \frac{L^2/L_C^2}{\pi^2 n(n+1)+L^2/L_C^2}Q_n\right]\mathrm{e}^{-[\pi^2 n(n+1)+L^2/L_C^2]\tau}\right\} \times P_n\{\cos\pi\varphi'\} \quad (20)$$

where

$$P_n(x) = \frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}[(x^2-1)^n]$$

are the Legendre polynomials which are the eigenfunctions of the corresponding Sturm–Liouville problem

$$Q_n = \frac{(2n+1)\pi}{2}\int_0^1 q_s^*(\varphi')P_n\{\cos\pi\varphi'\}\sin\pi\varphi'\,\mathrm{d}\varphi'$$

are the coefficients of the eigenfunction expansion of the dimensionless surface heat flux distribution

$$\theta_{n0} = \frac{(2n+1)\pi}{2}\theta_0\int_0^1 P_n\{\cos\pi\varphi'\}\sin\pi\varphi'\,\mathrm{d}\varphi'$$

are the coefficients of the eigenfunction expansion of the dimensionless initial temperature and $L = \pi R$ is the semicircumferential length of the shell.

For the values $\tau \gg 1$ the expression (20) gives the stationary solution

$$\theta(\varphi') = \frac{\pi}{2}\int_0^1 q_s^*(\varphi')\sin\pi\varphi'\,\mathrm{d}\varphi' + \sum_{n=1}^{\infty} Q_n\frac{L^2/L_C^2}{\pi^2 n(n+1)+L^2/L_C^2}P_n\{\cos\pi\varphi'\} \quad (21)$$

which includes two components. The first term presents the mean value of the source function. The second term is given by the infinite series dependent on the dimensionless criterion L/L_C.

3.4. *The criterion limits*

Now let us analyse the influence of the conductance on the temperature fields of the shells. In Figs. 2(a) and (b) the distributions of the dimensionless temperature in the shells for the different values of the criterion L/L_C are shown. For the shown graphs the dimensionless surface heat flux distribution is given by the following function:

$$q_s^*(\varphi') = \begin{cases}\cos\pi\varphi', & 0 \leqslant \varphi' < 0.5\\ 0, & 0.5 \leqslant \varphi' < 1\end{cases} \quad (22)$$

which corresponds to the case of the direct solar radi-

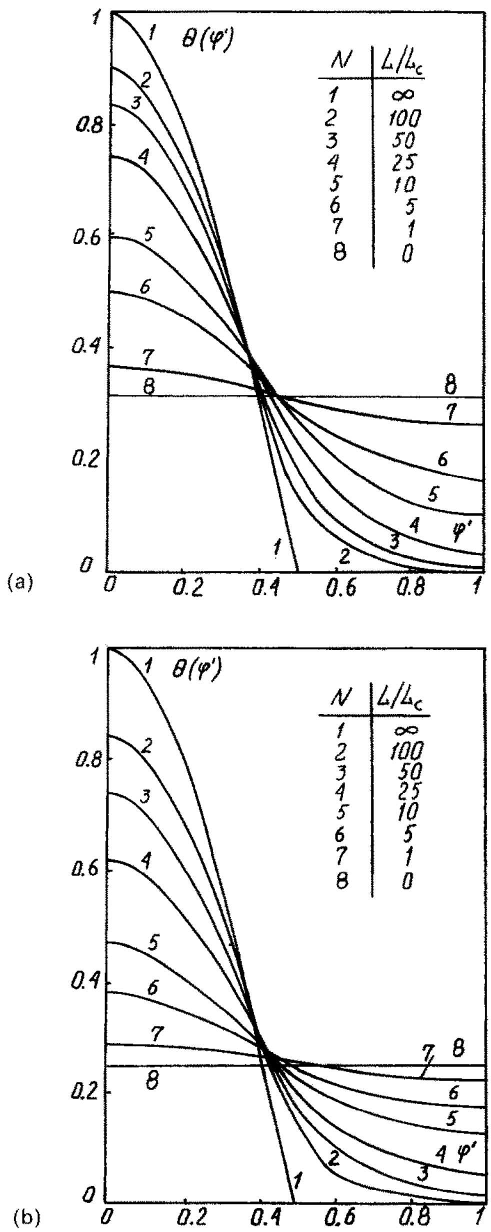

FIG. 2. Dimensionless temperature fields in the shells for different values of L/L_C. (a) Cylindrical shell; (b) spherical shell.

ation heating with the maximal value in the point $\varphi' = 0$.

The graphs show that in the limit $L/L_C \to 0$ the shells are isothermal. Their temperature results from the total balance between the fluxes of energy. The temperature for this case is given by the first term of equations (19) and (21). In this limit the contribution of the conductance presented by the infinite series is small and can be neglected. For the cylindrical shell the temperature in the limit $L/L_C \to 0$ is given by

$$\theta_1(\varphi') = \pi^{-1}, \quad T_1 = T_\alpha^* + q_s(0)/\pi h_s. \tag{23}$$

For the spherical shell the temperature for this operating limit is given by

$$\theta_1(\varphi') = 4^{-1}, \quad T_1 = T_\alpha^* + q_s(0)/4h_s. \tag{24}$$

For values of the criterion $L/L_C \approx 1$ the non-uniformity of the temperature field becomes considerable. The further increase in the criterion L/L_C leads to the increasing of the temperature non-uniformity. For values of $L/L_C > 10^2$ the temperature field approaches, and in the limit $L/L_C \to \infty$, coincides with the function:

$$\theta_\alpha(\varphi') = \begin{cases} \cos \pi\varphi', & 0 \leqslant \varphi' < 0.5 \\ 0, & 0.5 \leqslant \varphi' < 1 \end{cases}. \tag{25}$$

For this operating limit the temperature field results from the local balance between the heat fluxes on both surfaces of the shell and is not influenced by the conductance.

So, the completed parametric study of the solutions shows that two limit cases take place. For the first one the temperature field of the shells can be determined from the integral balance of energy fluxes. For the second one the temperature field of the shells can be determined from the local balance of the surface heat flux densities. We carried out evaluations of the criterion L/L_C which limit the applicability of both the limit approximations of the heat transfer process. The limit values of L/L_C were obtained from the analysis of the following relative differences between the exact solutions (19), (21); the approximations (23), (24); and (25):

$$\Delta = \max \left| \frac{T - T_1}{T - T_\alpha^*} \right| \tag{26}$$

where T is the exact solution of the problem and T_1 is the approximation of the problem.

For the characteristic value of the relative difference between the exact solutions and the approximations the value $\Delta = 10^{-2}$ was used. For the cylindrical shell the following limit values of the criterion L/L_C are obtained. For the values $L/L_C \leqslant 5 \times 10^{-2}$ the temperature field of the shell can be determined from the integral balance of heat fluxes. For values $L/L_C > 10^3$ the temperature field can be determined from the local balance of the surface heat flux densities without introducing the conductivity into the model. The relative errors of the temperature field approximations do not exceed a 1% value for the mentioned relations. The problem of the temperature field in the cylindrical shells should be formulated in the form of the conductivity equation for the values of the criterion lying within the following interval:

$$5 \times 10^{-2} < L/L_C < 10^3.$$

For the case of the spherical shells the integral balance of heat fluxes gives a sufficiently accurate approximation for the values $L/L_C < 1.3 \times 10^{-1}$. The local heat fluxes balance approximation is valid for criterion values of $L/L_C > 8 \times 10^3$. The maximum error of the approximations does not exceed 1%. The temperature field must be considered on the basis of the conductivity equation model only for the following interval of values:

$$1.3 \times 10^{-1} < L/L_C < 8 \times 10^3.$$

4. THE TEMPERATURE FIELD MODELS IN TRUSS CONSTRUCTIONS

Let us consider the temperature fields in the truss constructions as another important example of applicability of the introduced criterion. The temperature field models of the truss constructions under non-uniform heat exchange conditions are of great interest for the development of precise radio telescopes [1, 2, 7, 8]. This interest is explained by the considerable thermal deformations, resulting from the temperature non-uniformity in the truss constructions supporting the radio-wave reflectors. These deformations alter the reflecting surface accuracy and the efficiency of the antenna [1].

The radio telescopes truss constructions have many rod-like conjuncted elements oriented differently in space. The antenna truss constructions operate under solar irradiation, the infrared radiative fluxes of the atmosphere, earth and other neighbouring surfaces. Thermal losses occur by convective heat transfer and radiation. As a result of the different space orientations, the mentioned radiative fluxes and convective flow, respectively, different heat transfer conditions are formed on the surface of the truss elements. The truss elements are conjuncted and, therefore, conductive heat transfer takes place between them through the conjuctions. So, at the stage of the formulation of the heat transfer problem it is important to know in advance the influence of these conductive fluxes on the temperature fields of the truss elements.

For the purpose of the evaluation of the conductivity contribution in the temperature field models let us consider the problem of the conductive interaction between the semi-infinite rods through the conjunction, as shown in Fig. 3(a). As a result of the different orientations of the rods the different surface heat fluxes and convective heat transfer coefficient takes place for them. The described model problem for the low Biot number rods can be formulated in the following two equations:

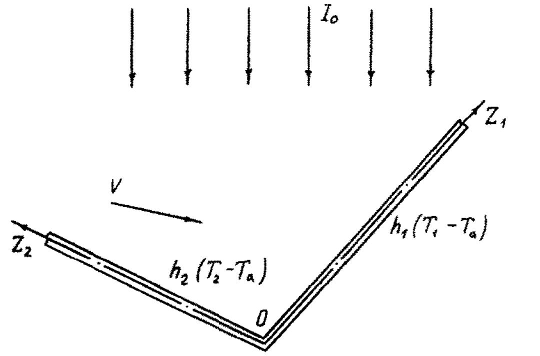

FIG. 3(a). Scheme for the study of conductive interaction between the truss elements.

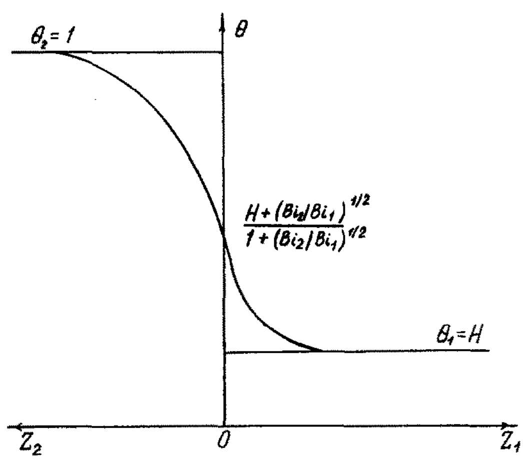

FIG. 3(b). Dimensionless temperature field in the truss elements for the case $H < 1$.

$$\delta'_1 k_1 \frac{d^2 T_1}{dz_1^2} = h'_1(T_1 - T_\infty) - q_1, \quad 0 \leqslant z_1 < \infty \quad (27)$$

$$\delta'_2 k_2 \frac{d^2 T_2}{dz_2^2} = h'_2(T_2 - T_\infty) - q_2, \quad 0 \leqslant z_2 < \infty \quad (28)$$

with the boundary conditions in the point of the conjunction

$$T_1|_{z=0} = T_2|_{z=0}, \quad \left.\frac{dT_1}{dz_1}\right|_{z=0} = \left.\frac{dT_2}{dz_2}\right|_{z=0} \quad (29)$$

and with the following conditions in the limit $z \to \infty$

$$\left.\frac{dT_1}{dz_1}\right|_{z_1\to\infty} = 0, \quad \left.\frac{dT_2}{dz_2}\right|_{z_2\to\infty} = 0 \quad (30)$$

where $\delta' = S/P$ are the ratios of the cross-sectional areas to the heat transfer perimeters of the roads, h'_1, h'_2 are the convective heat transfer coefficients for both rods, q_1, q_2 are the surface heat flux densities for both rods.

To present the solution more compactly one can introduce the following dimensionless independent variables:

$$\zeta_1 = z_1/\delta'_1, \quad \zeta_2 = z_2/\delta'_2$$

and the dimensionless temperatures

$$\theta_1 = (T_1 - T_\infty)h_2/q_2, \quad \theta_2 = (T_2 - T_\infty)h_2/q_2.$$

The solution of this problem is given by the expressions

$$\theta_1(\zeta_1) = H + \frac{(1-H)Bi_2^{1/2}\,Bi_1^{-1/2}}{1+Bi_2^{1/2}\,Bi_1^{-1/2}} \exp(-\sqrt{Bi_1}\,\zeta_1) \quad (31)$$

$$\theta_2(\zeta_2) = 1 + \frac{H-1}{1+Bi_2^{1/2}\,Bi_1^{-1/2}} \exp(-\sqrt{Bi_2}\,\zeta_2) \quad (32)$$

where $Bi_1 = h_1\delta'_1/k_1$, $Bi_2 = h_2\delta'_2/k_2$ are the Biot numbers for both rods, $H = q_1h_2/q_2h_1$ is the dimensionless complex including heat flux densities and heat transfer coefficients.

In Fig. 3(b) the dimensionless distribution for the value $H < 1$ is shown. The graph shows that the conductivity interaction penetrates from the conjunction point into the rods on the limited lengths. Beyond these lengths the temperatures of the rods are given by the values resulting from the surface heat transfer balances. For the evaluation of the conductivity interaction lengths expression (9) can be used. Substituting (31) and (32) into (9) one can obtain the following evaluations:

$$\delta_{T1} = (\delta'_1 k_1/h_1)^{1/2} = L_{C_1} \quad (34)$$

$$\delta_{T2} = (\delta'_2 k_2/h_2)^{1/2} = L_{C_2} \quad (35)$$

which coincide completely with the conductive length introduced for the case of the thermally-thin shells. So, the conductivity interaction through the conjunction does not penetrate into the rods beyond the conductive lengths, expressed through the value of the ratio of the cross-sectional areas to the heat transfer perimeters $\delta' = S/P$.

When the conductive lengths are much smaller than the geometrical lengths of the elements:

$$L_i/L_{C_i} \gg 1, \quad i = 1, \ldots, N \quad (36)$$

the conductance contribution and the conductive interaction between the elements can be neglected in the heat transfer model and the temperatures of the elements can be approximated by the following evaluations:

$$T_i = T_\infty + q_i/h_i \quad (37)$$

which result from the heat transfer balances on the surfaces of the elements.

In the case when the relations in equation (36) are not fulfilled, the conductive interactions of the elements are considerable and the temperatures along the elements are not uniform. In this case the temperature distribution along the elements can be described by the solution of the second order differential equations with the equivalent heat sources in the following form:

$$\theta_i(\zeta_i) = H_i + C_{1i}\exp(-\sqrt{Bi_i}\,\zeta_i) + C_{2i}\exp(-\sqrt{Bi_i}\,\zeta_i), \quad i = 1,\ldots,N \quad (38)$$

where $\zeta_i = z_i/\delta_i'$ are the longitudinal dimensionless coordinates of the elements, $Bi_i = \delta_i' h_i/k_i$ are the Biot numbers of the elements, $H_i = q_i h^*/h_i q^*$ are the dimensionless complexes, h^*, q^* is the pair of characteristic values. If h^*, q^* corresponds to the maximal value from the series q_i/h_i, $i = 1,\ldots,N$, all the dimensionless temperature distributions lie within the unit interval [0, 1]. The integration constants C_{1i}, C_{1i} of the expression (38) can be obtained by solving the system of the algebraic equations, resulting from the substitution of the expressions (38) into the boundary conditions for the elements conjunctions in the considered truss construction.

Thus, the question of the inclusion of the conductivity in the models of truss construction temperature regimes can also be considered with the help of the introduced conductive length and the criterion which is given by the ratio of the geometrical lengths of the elements to their conductive lengths.

5. CONCLUSIONS AND SUMMARY

Many heat transfer problems of thermally-thin shells and truss constructions can be formulated in the form of the second order differential conductivity equation where the surface heat transfer conditions are presented by the equivalent energy sources. This approach is valid for the low Biot number shells and truss constructions. For some operating conditions the conductance contribution in the temperature field can be neglected. In this case the second order differential component $k\nabla^2 T$ can be neglected in the equation for the temperature field which considerably simplifies the models. The contribution of conductivity in the heat transfer problems can be evaluated on the basis of the introduced conductive length which limits the spatial influence of the conductive heat transfer. Physically this characteristic presents the length within which the conductive heat transfer flux is dissipated by the heat exchange fluxes on the surfaces of the shell. So, the conductive heat transfer fluxes resulting from the non-uniformity of temperature in the shell have an effect only within the distances of the conductive length.

The introduction of this characteristic allowed us to obtain a simple criterion for the consideration of the conductivity contribution into the temperature fields of the thermally-thin shells and truss constructions. This criterion is presented by the ratio of the characteristic geometrical length to the conductivity length. In cases when this criterion is much larger than unity the conductance contribution can be neglected. Its influence is considerable only along the boundaries of the stepwise changes of heat transfer conditions on the surfaces. This influence affects the temperature fields only within the distance of the conductive length. Beyond these distances the temperature fields are determined only by heat transfer balances on the surfaces. In the case when the above mentioned relation for the introduced criterion is not fulfilled the conductance contribution is considerable and should be taken into account. For values of the criterion which are much smaller than unity the conductive heat transfer contribution is so high that even under non-uniform heat transfer conditions on the surfaces it would make uniform the temperature fields of the shells and truss elements.

The characteristics introduced in this paper allow one to estimate the influence of conductive heat transfer on the temperature fields at the stage of the formulation of heat transfer models for the operation of thermally-thin shells and truss constructions.

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LA LONGUEUR CONDUCTIVE EN PROBLEMES DE L'ECHANGE THERMIQUE POUR LES ENVELOPPES ET LES CONSTRUCTIONS DES FERMES

Résumé—L'analyse de l'influence de conductance aux champs de température en les enveloppes et les fermes, supposés thermiquement minces, est fait. Pour le limit spacial de l'influence de conductance la longueur conductive est introduite. Pour l'évaluation de la contribution de conductance le critérium, presenté par le rapport de dimension characteristique envers la longueur conductive est proposé. La conductance est négligeable pour les valeurs du critérium, lesquelles sont beaucoup moins de l'unité. Pour les cas quand cette relation est violée la contribution de la conductance est considérable. L'évaluation de critérium proposé permets de déterminer la nécessité de la conductance aux modèles de l'échange thermique au stage de la formulation des problèmes. Pour les examples de l'application du critérium proposé on peut mentionner le dévelopment des modèles de l'échange thermique pour les enveloppes de dirigeables, les enveloppes et les fermes de radiotélescopes.

DIE KONDUCTIVE LÄNGE IN PROBLEME DER WÄRMEARTBEITSWEISE DER AUSDEHNEN HÜLLEN UND TRÄGERENE AUFBAU

Zusammenfassung—In der Artikel fuhrtet man der Analyse der Einfluß der Wärmeleitfahigkeit an der Formierung der Wärmearbeitsweis der thermischfein Hüllen und ausdehnen trägerenen Aufbau durch. Man einfuhrtet die konduktive Länge, wie die Raumeinfluss der Wärmeleitfahigkeit begrenzt. Man einfuhrtet die Kriterium, wie der Verhaltnis der charakteren Raumabmessung zu der konduktiven Länge darstellen ist. Es ist moglich der Einfluß der Wärmeleitfahigkeit mißachten, wenn diese Kriterium zehr kleiner wie Eins ist. In der Fall, ween diese Bedingung ausfuhrt sich nicht, ist die Einfluß der Wärmeleitfahigkeit wesentlich. Die Schartung diese Kriterium in der Stadium Problemestellung erlaubt die Einfluß der Wärmeleitfahigkeit schatzenund und die Frage um die Notwendichkeit ihre Einschließung im model der Wärmeaustausch. Der Beispiel der Anwendung diese Kriterium and konductive Länge ist die Erarbeitung der Modele der Wärmeaustausch der Hülle der Zuftschwimmenapparate, Hulle und trägerene Aufbau der radiotelescope.

КОНДУКТИВНАЯ ДЛИНА В ЗАДАЧАХ ТЕПЛОВОГО РЕЖИМА ОБОЛОЧЕК И ФЕРМЕНТНЫХ КОНСТРУКЦИЙ

Аннотация—Анализируется влияние теплопроводности на температурные поля в термически тонких оболочках и ферментных конструкциях. В качестве пространственного предела влияния теплопроводндости в упомянутых задачах теплообмена вводится кондуктивная длина. Получен критерий, выраженный отношением характерного размера к кондуктивной длине. На основе задач теплообмена показано, что теплопроводностью можно пренебречь, если указаное отношение намного меньше единипы. В случае, когда это отношение не выполняется, вклад теплопроводности является существенным. Оценка данного критерия на стадии постановки задачи позволяет определить необходимую величину теплопроводности в моделях температурных полей. Применение введенного критерия может рассматриваться на вримере разработки моделей расчета теплообменников, оболочек радиотелескопов, а также ферментных конструкций.